

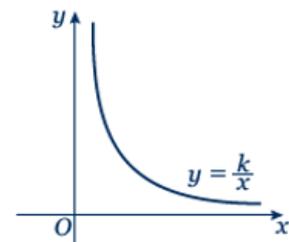
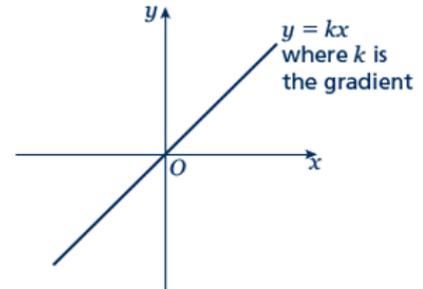
# Proportion

## A LEVEL LINKS

**Scheme of work:** 2a. Straight-line graphs, parallel/perpendicular, length and area problems

### Key points

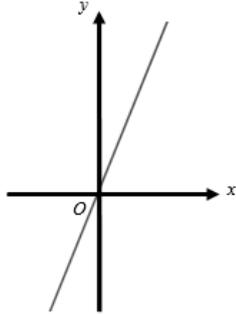
- Two quantities are in direct proportion when, as one quantity increases, the other increases at the same rate. Their ratio remains the same.
- 'y is directly proportional to x' is written as  $y \propto x$ .  
If  $y \propto x$  then  $y = kx$ , where  $k$  is a constant.
- When  $x$  is directly proportional to  $y$ , the graph is a straight line passing through the origin.
- Two quantities are in inverse proportion when, as one quantity increases, the other decreases at the same rate.
- 'y is inversely proportional to x' is written as  $y \propto \frac{1}{x}$ .  
If  $y \propto \frac{1}{x}$  then  $y = \frac{k}{x}$ , where  $k$  is a constant.
- When  $x$  is inversely proportional to  $y$  the graph is the same shape as the graph of  $y = \frac{1}{x}$



### Examples

- Example 1**  $y$  is directly proportional to  $x$ .  
When  $y = 16$ ,  $x = 5$ .
- Find  $x$  when  $y = 30$ .
  - Sketch the graph of the formula.

<p><b>a</b> <math>y \propto x</math></p> $y = kx$ $16 = k \times 5$ $k = 3.2$ $y = 3.2x$ <p>When <math>y = 30</math>,</p> $30 = 3.2 \times x$ $x = 9.375$	<ol style="list-style-type: none"> <li>Write <math>y</math> is directly proportional to <math>x</math>, using the symbol <math>\propto</math>.</li> <li>Write the equation using <math>k</math>.</li> <li>Substitute <math>y = 16</math> and <math>x = 5</math> into <math>y = kx</math>.</li> <li>Solve the equation to find <math>k</math>.</li> <li>Substitute the value of <math>k</math> back into the equation <math>y = kx</math>.</li> <li>Substitute <math>y = 30</math> into <math>y = 3.2x</math> and solve to find <math>x</math> when <math>y = 30</math>.</li> </ol>
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<p><b>b</b></p> 	<p><b>7</b> The graph of <math>y = 3.2x</math> is a straight line passing through <math>(0, 0)</math> with a gradient of 3.2.</p>
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**Example 2**  $y$  is directly proportional to  $x^2$ .  
When  $x = 3$ ,  $y = 45$ .

**a** Find  $y$  when  $x = 5$ .  
**b** Find  $x$  when  $y = 20$ .

<p><b>a</b> <math>y \propto x^2</math></p> $y = kx^2$ $45 = k \times 3^2$ $k = 5$ $y = 5x^2$ <p>When <math>x = 5</math>,</p> $y = 5 \times 5^2$ $y = 125$ <p><b>b</b> <math>20 = 5 \times x^2</math></p> $x^2 = 4$ $x = \pm 2$	<ol style="list-style-type: none"> <li><b>1</b> Write <math>y</math> is directly proportional to <math>x^2</math>, using the symbol <math>\propto</math>.</li> <li><b>2</b> Write the equation using <math>k</math>.</li> <li><b>3</b> Substitute <math>y = 45</math> and <math>x = 3</math> into <math>y = kx^2</math>.</li> <li><b>4</b> Solve the equation to find <math>k</math>.</li> <li><b>5</b> Substitute the value of <math>k</math> back into the equation <math>y = kx^2</math>.</li> <li><b>6</b> Substitute <math>x = 5</math> into <math>y = 5x^2</math> and solve to find <math>y</math> when <math>x = 5</math>.</li> <li><b>7</b> Substitute <math>y = 20</math> into <math>y = 5x^2</math> and solve to find <math>x</math> when <math>y = 20</math>.</li> </ol>
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**Example 3**  $P$  is inversely proportional to  $Q$ .  
When  $P = 100$ ,  $Q = 10$ .  
Find  $Q$  when  $P = 20$ .

$P \propto \frac{1}{Q}$ $P = \frac{k}{Q}$ $100 = \frac{k}{10}$ $k = 1000$ $P = \frac{1000}{Q}$ $20 = \frac{1000}{Q}$ $Q = \frac{1000}{20} = 50$	<ol style="list-style-type: none"> <li><b>1</b> Write <math>P</math> is inversely proportional to <math>Q</math>, using the symbol <math>\propto</math>.</li> <li><b>2</b> Write the equation using <math>k</math>.</li> <li><b>3</b> Substitute <math>P = 100</math> and <math>Q = 10</math>.</li> <li><b>4</b> Solve the equation to find <math>k</math>.</li> <li><b>5</b> Substitute the value of <math>k</math> into <math>P = \frac{k}{Q}</math>.</li> <li><b>6</b> Substitute <math>P = 20</math> into <math>P = \frac{1000}{Q}</math> and solve to find <math>Q</math> when <math>P = 20</math>.</li> </ol>
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## Practice

- 1 Paul gets paid an hourly rate. The amount of pay (£ $P$ ) is directly proportional to the number of hours ( $h$ ) he works.  
When he works 8 hours he is paid £56.  
If Paul works for 11 hours, how much is he paid?
- 2  $x$  is directly proportional to  $y$ .  
 $x = 35$  when  $y = 5$ .
  - a Find a formula for  $x$  in terms of  $y$ .
  - b Sketch the graph of the formula.
  - c Find  $x$  when  $y = 13$ .
  - d Find  $y$  when  $x = 63$ .
- 3  $Q$  is directly proportional to the square of  $Z$ .  
 $Q = 48$  when  $Z = 4$ .
  - a Find a formula for  $Q$  in terms of  $Z$ .
  - b Sketch the graph of the formula.
  - c Find  $Q$  when  $Z = 5$ .
  - d Find  $Z$  when  $Q = 300$ .
- 4  $y$  is directly proportional to the square of  $x$ .  
 $x = 2$  when  $y = 10$ .
  - a Find a formula for  $y$  in terms of  $x$ .
  - b Sketch the graph of the formula.
  - c Find  $x$  when  $y = 90$ .
- 5  $B$  is directly proportional to the square root of  $C$ .  
 $C = 25$  when  $B = 10$ .
  - a Find  $B$  when  $C = 64$ .
  - b Find  $C$  when  $B = 20$ .
- 6  $C$  is directly proportional to  $D$ .  
 $C = 100$  when  $D = 150$ .  
Find  $C$  when  $D = 450$ .
- 7  $y$  is directly proportional to  $x$ .  
 $x = 27$  when  $y = 9$ .  
Find  $x$  when  $y = 3.7$ .
- 8  $m$  is proportional to the cube of  $n$ .  
 $m = 54$  when  $n = 3$ .  
Find  $n$  when  $m = 250$ .

### Hint

Substitute the values given for  $P$  and  $h$  into the formula to calculate  $k$ .

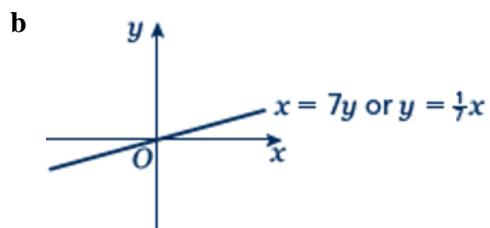
## Extend

- 9**  $s$  is inversely proportional to  $t$ .
- Given that  $s = 2$  when  $t = 2$ , find a formula for  $s$  in terms of  $t$ .
  - Sketch the graph of the formula.
  - Find  $t$  when  $s = 1$ .
- 10**  $a$  is inversely proportional to  $b$ .  
 $a = 5$  when  $b = 20$ .
- Find  $a$  when  $b = 50$ .
  - Find  $b$  when  $a = 10$ .
- 11**  $v$  is inversely proportional to  $w$ .  
 $w = 4$  when  $v = 20$ .
- Find a formula for  $v$  in terms of  $w$ .
  - Sketch the graph of the formula.
  - Find  $w$  when  $v = 2$ .
- 12**  $L$  is inversely proportional to  $W$ .  
 $L = 12$  when  $W = 3$ .  
Find  $W$  when  $L = 6$ .
- 13**  $s$  is inversely proportional to  $t$ .  
 $s = 6$  when  $t = 12$ .
- Find  $s$  when  $t = 3$ .
  - Find  $t$  when  $s = 18$ .
- 14**  $y$  is inversely proportional to  $x^2$ .  
 $y = 4$  when  $x = 2$ .  
Find  $y$  when  $x = 4$ .
- 15**  $y$  is inversely proportional to the square root of  $x$ .  
 $x = 25$  when  $y = 1$ .  
Find  $x$  when  $y = 5$ .
- 16**  $a$  is inversely proportional to  $b$ .  
 $a = 0.05$  when  $b = 4$ .
- Find  $a$  when  $b = 2$ .
  - Find  $b$  when  $a = 2$ .

## Answers

1 £77

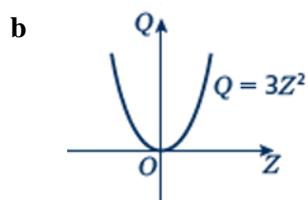
2 a  $x = 7y$



c 91

d 9

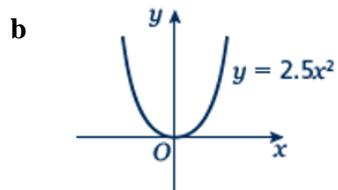
3 a  $Q = 3Z^2$



c 75

d  $\pm 10$

4 a  $y = 2.5x^2$



c  $\pm 6$

5 a 16

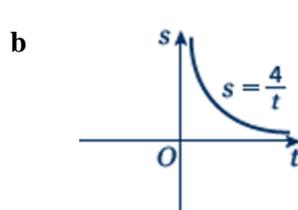
b 100

6 300

7 11.1

8 5

9 a  $s = \frac{4}{t}$

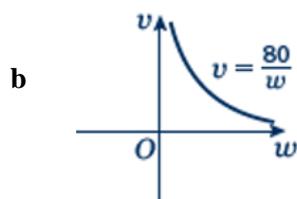


c 4

10 a 2

b 10

11 a  $v = \frac{80}{w}$



c 40

12 6

13 a 24

b 4

14 1

15 1

16 a 0.1

b 0.1