

Translating graphs

A LEVEL LINKS

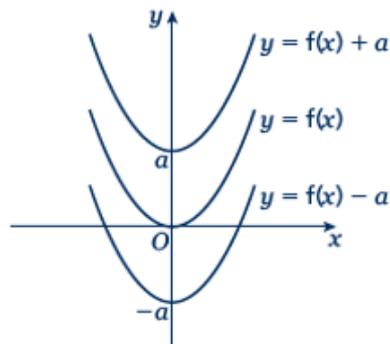
Scheme of work: 1f. Transformations – transforming graphs – $f(x)$ notation

Key points

- The transformation $y = f(x) \pm a$ is a translation of $y = f(x)$ parallel to the y -axis; it is a vertical translation.

As shown on the graph,

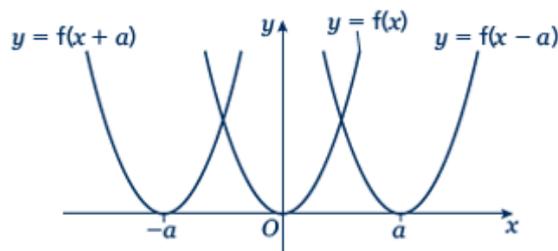
- $y = f(x) + a$ translates $y = f(x)$ up
- $y = f(x) - a$ translates $y = f(x)$ down.



- The transformation $y = f(x \pm a)$ is a translation of $y = f(x)$ parallel to the x -axis; it is a horizontal translation.

As shown on the graph,

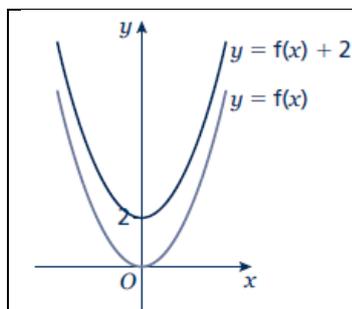
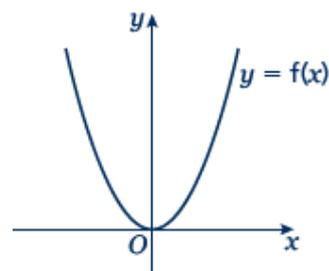
- $y = f(x + a)$ translates $y = f(x)$ to the left
- $y = f(x - a)$ translates $y = f(x)$ to the right.



Examples

Example 1 The graph shows the function $y = f(x)$.

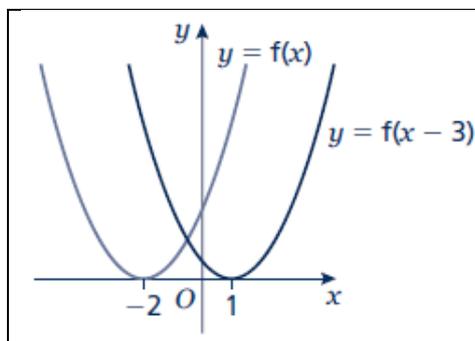
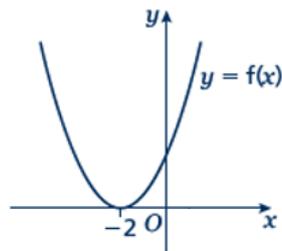
Sketch the graph of $y = f(x) + 2$.



For the function $y = f(x) + 2$ translate the function $y = f(x)$ 2 units up.

Example 2 The graph shows the function $y = f(x)$.

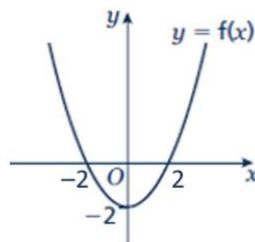
Sketch the graph of $y = f(x - 3)$.



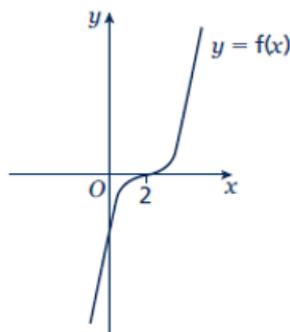
For the function $y = f(x - 3)$ translate the function $y = f(x)$ 3 units right.

Practice

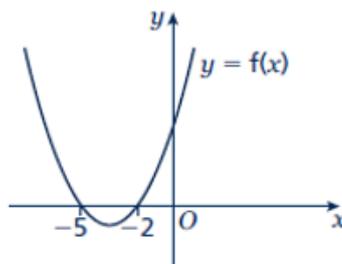
- 1** The graph shows the function $y = f(x)$. Copy the graph and on the same axes sketch and label the graphs of $y = f(x) + 4$ and $y = f(x + 2)$.



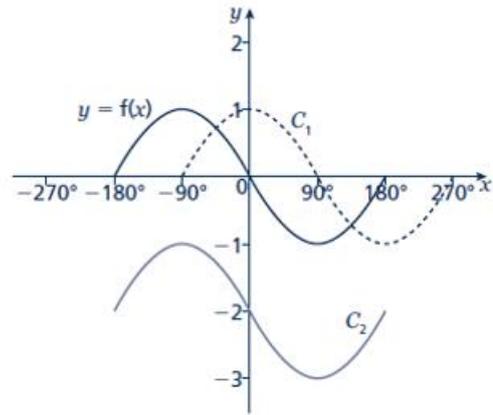
- 2** The graph shows the function $y = f(x)$. Copy the graph and on the same axes sketch and label the graphs of $y = f(x + 3)$ and $y = f(x) - 3$.



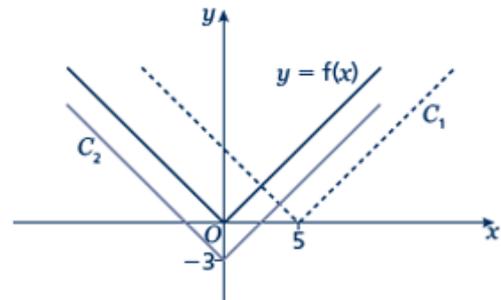
- 3** The graph shows the function $y = f(x)$. Copy the graph and on the same axes sketch the graph of $y = f(x - 5)$.



- 4 The graph shows the function $y = f(x)$ and two transformations of $y = f(x)$, labelled C_1 and C_2 . Write down the equations of the translated curves C_1 and C_2 in function form.

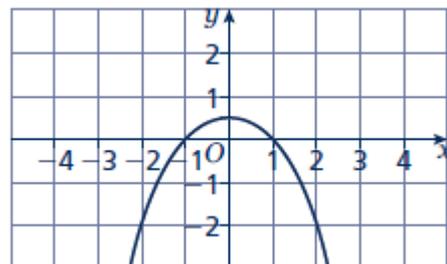


- 5 The graph shows the function $y = f(x)$ and two transformations of $y = f(x)$, labelled C_1 and C_2 . Write down the equations of the translated curves C_1 and C_2 in function form.



- 6 The graph shows the function $y = f(x)$.

- a Sketch the graph of $y = f(x) + 2$
- b Sketch the graph of $y = f(x + 2)$



Stretching graphs

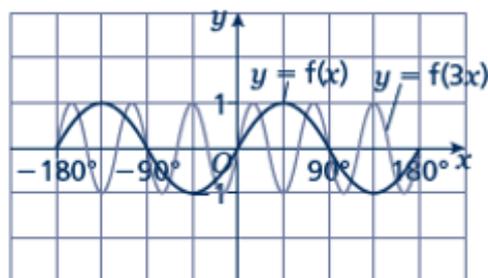
A LEVEL LINKS

Scheme of work: 1f. Transformations – transforming graphs – $f(x)$ notation

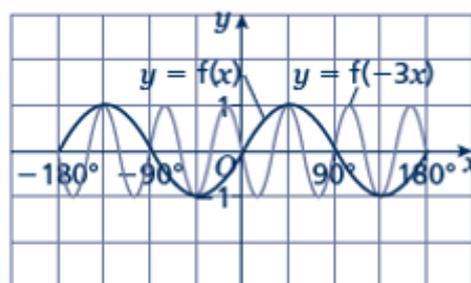
Textbook: Pure Year 1, 4.6 Stretching graphs

Key points

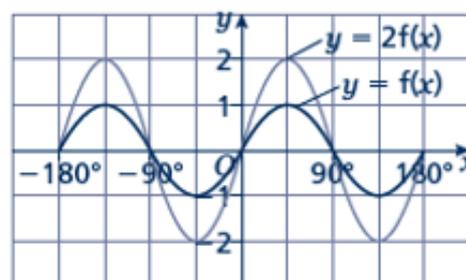
- The transformation $y = f(ax)$ is a horizontal stretch of $y = f(x)$ with scale factor $\frac{1}{a}$ parallel to the x -axis.



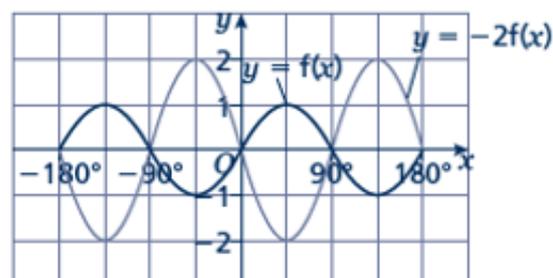
- The transformation $y = f(-ax)$ is a horizontal stretch of $y = f(x)$ with scale factor $\frac{1}{a}$ parallel to the x -axis and then a reflection in the y -axis.



- The transformation $y = af(x)$ is a vertical stretch of $y = f(x)$ with scale factor a parallel to the y -axis.



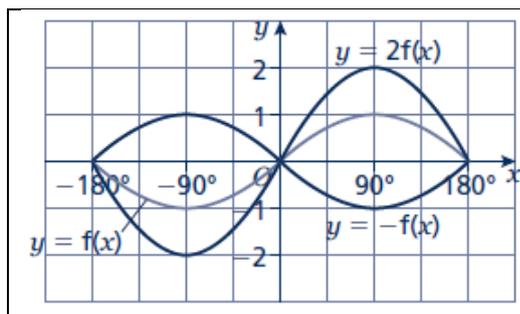
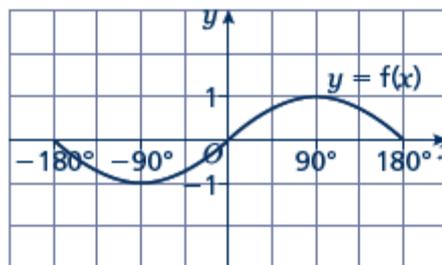
- The transformation $y = -af(x)$ is a vertical stretch of $y = f(x)$ with scale factor a parallel to the y -axis and then a reflection in the x -axis.



Examples

Example 3 The graph shows the function $y = f(x)$.

Sketch and label the graphs of $y = 2f(x)$ and $y = -f(x)$.

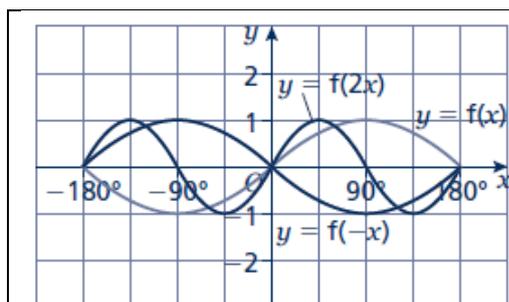
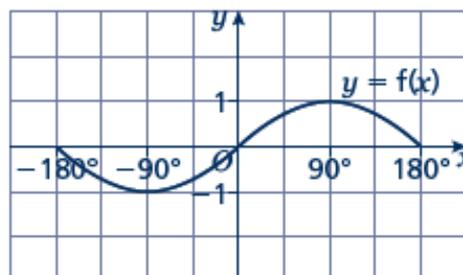


The function $y = 2f(x)$ is a vertical stretch of $y = f(x)$ with scale factor 2 parallel to the y -axis.

The function $y = -f(x)$ is a reflection of $y = f(x)$ in the x -axis.

Example 4 The graph shows the function $y = f(x)$.

Sketch and label the graphs of $y = f(2x)$ and $y = f(-x)$.

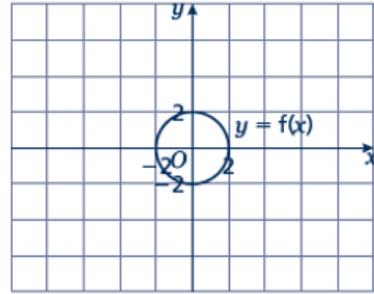


The function $y = f(2x)$ is a horizontal stretch of $y = f(x)$ with scale factor $\frac{1}{2}$ parallel to the x -axis.

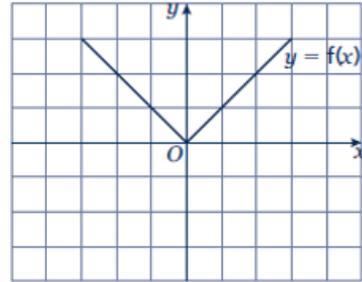
The function $y = f(-x)$ is a reflection of $y = f(x)$ in the y -axis.

Practice

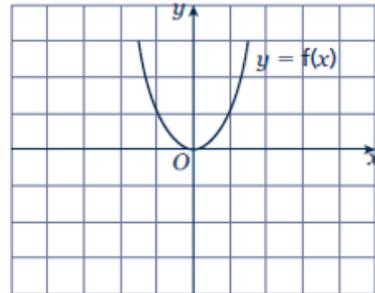
- 7 The graph shows the function $y = f(x)$.
- Copy the graph and on the same axes sketch and label the graph of $y = 3f(x)$.
 - Make another copy of the graph and on the same axes sketch and label the graph of $y = f(2x)$.



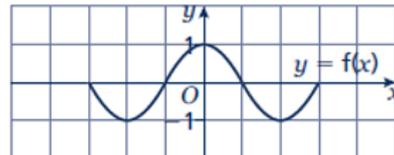
- 8 The graph shows the function $y = f(x)$. Copy the graph and on the same axes sketch and label the graphs of $y = -2f(x)$ and $y = f(3x)$.



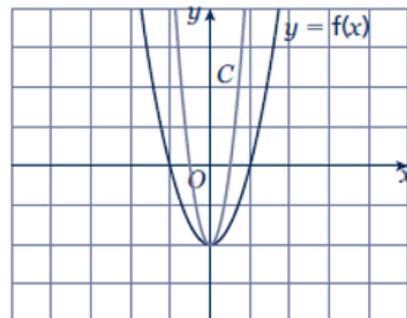
- 9 The graph shows the function $y = f(x)$. Copy the graph and, on the same axes, sketch and label the graphs of $y = -f(x)$ and $y = f\left(\frac{1}{2}x\right)$.



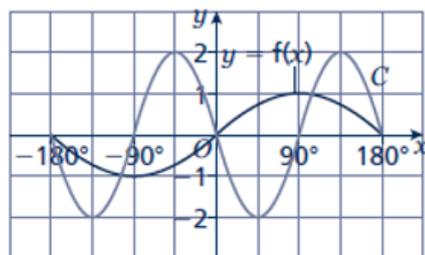
- 10 The graph shows the function $y = f(x)$. Copy the graph and, on the same axes, sketch the graph of $y = -f(2x)$.



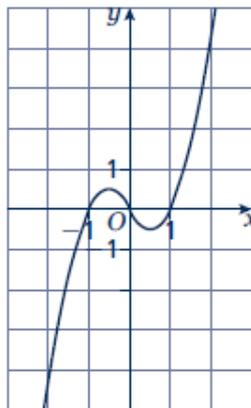
- 11 The graph shows the function $y = f(x)$ and a transformation, labelled C . Write down the equation of the translated curve C in function form.



- 12 The graph shows the function $y = f(x)$ and a transformation labelled C . Write down the equation of the translated curve C in function form.



- 13 The graph shows the function $y = f(x)$.
- Sketch the graph of $y = -f(x)$.
 - Sketch the graph of $y = 2f(x)$.

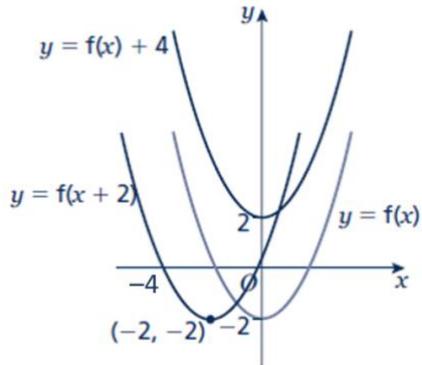


Extend

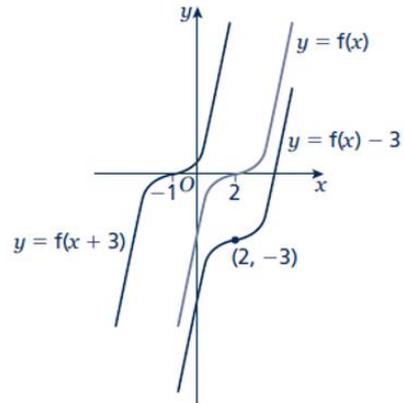
- 14
- Sketch and label the graph of $y = f(x)$, where $f(x) = (x - 1)(x + 1)$.
 - On the same axes, sketch and label the graphs of $y = f(x) - 2$ and $y = f(x + 2)$.
- 15
- Sketch and label the graph of $y = f(x)$, where $f(x) = -(x + 1)(x - 2)$.
 - On the same axes, sketch and label the graph of $y = f\left(-\frac{1}{2}x\right)$.

Answers

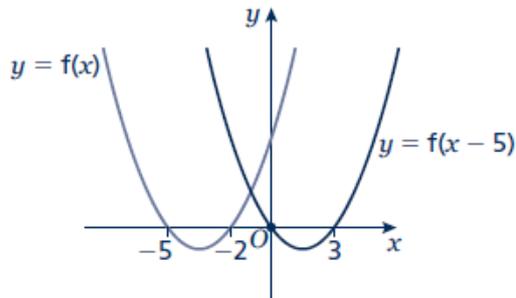
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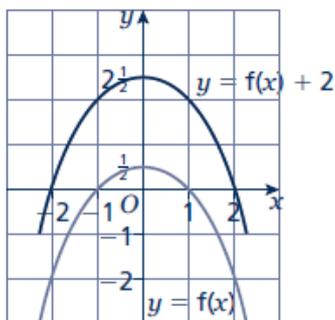
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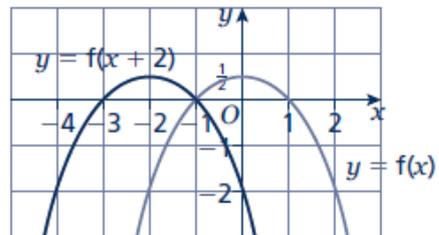
4 $C_1: y = f(x - 90^\circ)$
 $C_2: y = f(x) - 2$

5 $C_1: y = f(x - 5)$
 $C_2: y = f(x) - 3$

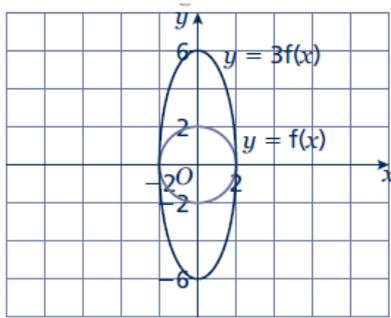
6 a



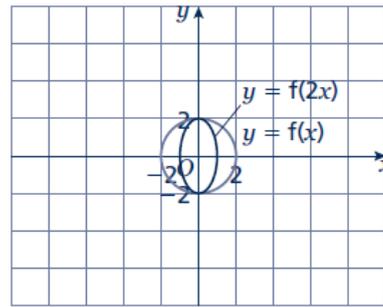
b



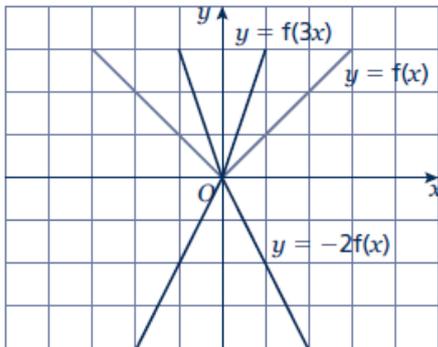
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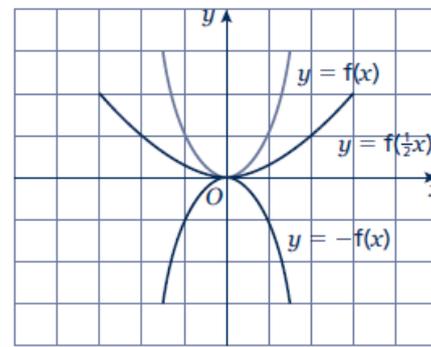
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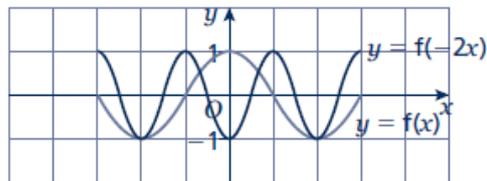
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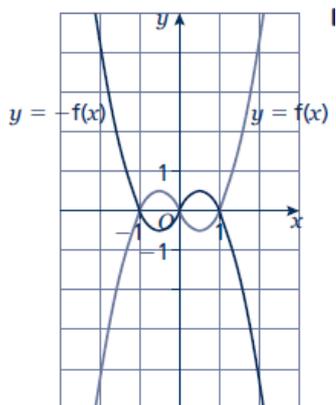
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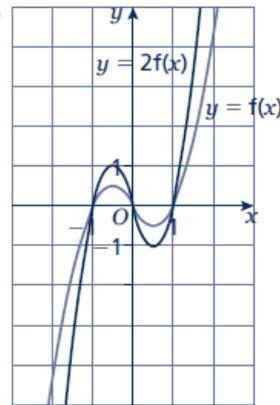
11 $y = f(2x)$

12 $y = -2f(2x)$ or $y = 2f(-2x)$

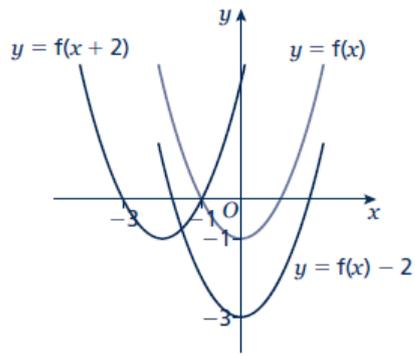
13 a



b



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