

Solving linear simultaneous equations using the elimination method

A LEVEL LINKS

Scheme of work: 1c. Equations – quadratic/linear simultaneous

Key points

- Two equations are simultaneous when they are both true at the same time.
- Solving simultaneous linear equations in two unknowns involves finding the value of each unknown which works for both equations.
- Make sure that the coefficient of one of the unknowns is the same in both equations.
- Eliminate this equal unknown by either subtracting or adding the two equations.

Examples

Example 1 Solve the simultaneous equations $3x + y = 5$ and $x + y = 1$

$\begin{array}{r} 3x + y = 5 \\ - \quad x + y = 1 \\ \hline 2x \quad = 4 \end{array}$ <p>So $x = 2$</p> <p>Using $x + y = 1$ $2 + y = 1$ So $y = -1$</p> <p>Check: equation 1: $3 \times 2 + (-1) = 5$ YES equation 2: $2 + (-1) = 1$ YES</p>	<ol style="list-style-type: none"> 1 Subtract the second equation from the first equation to eliminate the y term. 2 To find the value of y, substitute $x = 2$ into one of the original equations. 3 Substitute the values of x and y into both equations to check your answers.
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Example 2 Solve $x + 2y = 13$ and $5x - 2y = 5$ simultaneously.

$\begin{array}{r} x + 2y = 13 \\ + \quad 5x - 2y = 5 \\ \hline 6x \quad = 18 \end{array}$ <p>So $x = 3$</p> <p>Using $x + 2y = 13$ $3 + 2y = 13$ So $y = 5$</p> <p>Check: equation 1: $3 + 2 \times 5 = 13$ YES equation 2: $5 \times 3 - 2 \times 5 = 5$ YES</p>	<ol style="list-style-type: none"> 1 Add the two equations together to eliminate the y term. 2 To find the value of y, substitute $x = 3$ into one of the original equations. 3 Substitute the values of x and y into both equations to check your answers.
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Example 3 Solve $2x + 3y = 2$ and $5x + 4y = 12$ simultaneously.

$\begin{array}{r} (2x + 3y = 2) \times 4 \rightarrow 8x + 12y = 8 \\ (5x + 4y = 12) \times 3 \rightarrow \underline{15x + 12y = 36} \\ 7x = 28 \end{array}$ <p>So $x = 4$</p> <p>Using $2x + 3y = 2$ $2 \times 4 + 3y = 2$ $8 + 3y = 2$ $3y = 2 - 8$ $3y = -6$ $y = -2$</p> <p>Check: equation 1: $2 \times 4 + 3 \times (-2) = 2$ YES equation 2: $5 \times 4 + 4 \times (-2) = 12$ YES</p>	<ol style="list-style-type: none"> 1 Multiply the first equation by 4 and the second equation by 3 to make the coefficient of y the same for both equations. Then subtract the first equation from the second equation to eliminate the y term. 2 To find the value of y, substitute $x = 4$ into one of the original equations. 3 Substitute the values of x and y into both equations to check your answers.
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Practice

Solve these simultaneous equations.

1 $4x + y = 8$
 $x + y = 5$

2 $3x + y = 7$
 $3x + 2y = 5$

3 $4x + y = 3$
 $3x - y = 11$

4 $3x + 4y = 7$
 $x - 4y = 5$

5 $2x + y = 11$
 $x - 3y = 9$

6 $2x + 3y = 11$
 $3x + 2y = 4$

Solving linear simultaneous equations using the substitution method

A LEVEL LINKS

Scheme of work: 1c. Equations – quadratic/linear simultaneous

Textbook: Pure Year 1, 3.1 Linear simultaneous equations

Key points

- The substitution method is the method most commonly used for A level. This is because it is the method used to solve linear and quadratic simultaneous equations.

Examples

Example 4 Solve the simultaneous equations $y = 2x + 1$ and $5x + 3y = 14$

$5x + 3(2x + 1) = 14$ $5x + 6x + 3 = 14$ $11x + 3 = 14$ $11x = 11$ <p>So $x = 1$</p> <p>Using $y = 2x + 1$ $y = 2 \times 1 + 1$</p> <p>So $y = 3$</p> <p>Check: equation 1: $3 = 2 \times 1 + 1$ YES equation 2: $5 \times 1 + 3 \times 3 = 14$ YES</p>	<ol style="list-style-type: none"> Substitute $2x + 1$ for y into the second equation. Expand the brackets and simplify. Work out the value of x. To find the value of y, substitute $x = 1$ into one of the original equations. Substitute the values of x and y into both equations to check your answers.
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Example 5 Solve $2x - y = 16$ and $4x + 3y = -3$ simultaneously.

$y = 2x - 16$ $4x + 3(2x - 16) = -3$ $4x + 6x - 48 = -3$ $10x - 48 = -3$ $10x = 45$ <p>So $x = 4\frac{1}{2}$</p> <p>Using $y = 2x - 16$ $y = 2 \times 4\frac{1}{2} - 16$</p> <p>So $y = -7$</p> <p>Check: equation 1: $2 \times 4\frac{1}{2} - (-7) = 16$ YES equation 2: $4 \times 4\frac{1}{2} + 3 \times (-7) = -3$ YES</p>	<ol style="list-style-type: none"> Rearrange the first equation. Substitute $2x - 16$ for y into the second equation. Expand the brackets and simplify. Work out the value of x. To find the value of y, substitute $x = 4\frac{1}{2}$ into one of the original equations. Substitute the values of x and y into both equations to check your answers.
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Practice

Solve these simultaneous equations.

7 $y = x - 4$
 $2x + 5y = 43$

8 $y = 2x - 3$
 $5x - 3y = 11$

9 $2y = 4x + 5$
 $9x + 5y = 22$

10 $2x = y - 2$
 $8x - 5y = -11$

11 $3x + 4y = 8$
 $2x - y = -13$

12 $3y = 4x - 7$
 $2y = 3x - 4$

13 $3x = y - 1$
 $2y - 2x = 3$

14 $3x + 2y + 1 = 0$
 $4y = 8 - x$

Extend

15 Solve the simultaneous equations $3x + 5y - 20 = 0$ and $2(x + y) = \frac{3(y - x)}{4}$.

Answers

1 $x = 1, y = 4$

2 $x = 3, y = -2$

3 $x = 2, y = -5$

4 $x = 3, y = -\frac{1}{2}$

5 $x = 6, y = -1$

6 $x = -2, y = 5$

7 $x = 9, y = 5$

8 $x = -2, y = -7$

9 $x = \frac{1}{2}, y = 3\frac{1}{2}$

10 $x = \frac{1}{2}, y = 3$

11 $x = -4, y = 5$

12 $x = -2, y = -5$

13 $x = \frac{1}{4}, y = 1\frac{3}{4}$

14 $x = -2, y = 2\frac{1}{2}$

15 $x = -2\frac{1}{2}, y = 5\frac{1}{2}$