# Completing the square

> **A LEVEL LINKS**
> **Scheme of work:** 1b. Quadratic functions – factorising, solving, graphs and the discriminants

## Key points

- Completing the square for a quadratic rearranges $ax^2 + bx + c$ into the form $p(x + q)^2 + r$
- If $a \neq 1$, then factorise using $a$ as a common factor.

## Examples

**Example 1**    Complete the square for the quadratic expression $x^2 + 6x - 2$

| | |
|---|---|
| $x^2 + 6x - 2$ <br><br> $= (x + 3)^2 - 9 - 2$ <br><br> $= (x + 3)^2 - 11$ | **1** Write $x^2 + bx + c$ in the form $$\left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$$ <br> **2** Simplify |

**Example 2**    Write $2x^2 - 5x + 1$ in the form $p(x + q)^2 + r$

| | |
|---|---|
| $2x^2 - 5x + 1$ <br><br><br><br> $= 2\left(x^2 - \frac{5}{2}x\right) + 1$ <br><br> $= 2\left[\left(x - \frac{5}{4}\right)^2 - \left(\frac{5}{4}\right)^2\right] + 1$ <br><br> $= 2\left(x - \frac{5}{4}\right)^2 - \frac{25}{8} + 1$ <br><br><br><br> $= 2\left(x - \frac{5}{4}\right)^2 - \frac{17}{8}$ | **1** Before completing the square write $ax^2 + bx + c$ in the form $$a\left(x^2 + \frac{b}{a}x\right) + c$$ <br> **2** Now complete the square by writing $x^2 - \frac{5}{2}x$ in the form $$\left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$$ <br> **3** Expand the square brackets – don't forget to multiply $\left(\frac{5}{4}\right)^2$ by the factor of 2 <br> **4** Simplify |

## Practice

**1**  Write the following quadratic expressions in the form $(x + p)^2 + q$

    **a**   $x^2 + 4x + 3$                 **b**   $x^2 - 10x - 3$

    **c**   $x^2 - 8x$                       **d**   $x^2 + 6x$

    **e**   $x^2 - 2x + 7$               **f**   $x^2 + 3x - 2$

**2**  Write the following quadratic expressions in the form $p(x + q)^2 + r$

    **a**   $2x^2 - 8x - 16$            **b**   $4x^2 - 8x - 16$

    **c**   $3x^2 + 12x - 9$           **d**   $2x^2 + 6x - 8$

**3**  Complete the square.

    **a**   $2x^2 + 3x + 6$               **b**   $3x^2 - 2x$

    **c**   $5x^2 + 3x$                 **d**   $3x^2 + 5x + 3$

## Extend

**4**  Write $(25x^2 + 30x + 12)$ in the form $(ax + b)^2 + c$.

## Answers

**1**  **a**  $(x + 2)^2 - 1$          **b**  $(x - 5)^2 - 28$

     **c**  $(x - 4)^2 - 16$          **d**  $(x + 3)^2 - 9$

     **e**  $(x - 1)^2 + 6$          **f**  $\left(x + \dfrac{3}{2}\right)^2 - \dfrac{17}{4}$

**2**  **a**  $2(x - 2)^2 - 24$          **b**  $4(x - 1)^2 - 20$

     **c**  $3(x + 2)^2 - 21$          **d**  $2\left(x + \dfrac{3}{2}\right)^2 - \dfrac{25}{2}$

**3**  **a**  $2\left(x + \dfrac{3}{4}\right)^2 + \dfrac{39}{8}$          **b**  $3\left(x - \dfrac{1}{3}\right)^2 - \dfrac{1}{3}$

     **c**  $5\left(x + \dfrac{3}{10}\right)^2 - \dfrac{9}{20}$          **d**  $3\left(x + \dfrac{5}{6}\right)^2 + \dfrac{11}{12}$

**4**  $(5x + 3)^2 + 3$