

# Rules of indices

## A LEVEL LINKS

**Scheme of work:** 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

### Key points

- $a^m \times a^n = a^{m+n}$
- $\frac{a^m}{a^n} = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $a^0 = 1$
- $a^{\frac{1}{n}} = \sqrt[n]{a}$  i.e. the  $n$ th root of  $a$
- $a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$
- $a^{-m} = \frac{1}{a^m}$
- The square root of a number produces two solutions, e.g.  $\sqrt{16} = \pm 4$ .

### Examples

**Example 1** Evaluate  $10^0$

$10^0 = 1$	Any value raised to the power of zero is equal to 1
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**Example 2** Evaluate  $9^{\frac{1}{2}}$

$9^{\frac{1}{2}} = \sqrt{9}$ $= 3$	Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$
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**Example 3** Evaluate  $27^{\frac{2}{3}}$

$27^{\frac{2}{3}} = (\sqrt[3]{27})^2$ $= 3^2$ $= 9$	<ol style="list-style-type: none"> <li>1 Use the rule <math>a^{\frac{m}{n}} = (\sqrt[n]{a})^m</math></li> <li>2 Use <math>\sqrt[3]{27} = 3</math></li> </ol>
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**Example 4** Evaluate  $4^{-2}$

$4^{-2} = \frac{1}{4^2}$ $= \frac{1}{16}$	<ol style="list-style-type: none"> <li>1 Use the rule <math>a^{-m} = \frac{1}{a^m}</math></li> <li>2 Use <math>4^2 = 16</math></li> </ol>
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**Example 5** Simplify  $\frac{6x^5}{2x^2}$

$\frac{6x^5}{2x^2} = 3x^3$	$6 \div 2 = 3$ and use the rule $\frac{a^m}{a^n} = a^{m-n}$ to give $\frac{x^5}{x^2} = x^{5-2} = x^3$
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**Example 6** Simplify  $\frac{x^3 \times x^5}{x^4}$

$\frac{x^3 \times x^5}{x^4} = \frac{x^{3+5}}{x^4} = \frac{x^8}{x^4}$ $= x^{8-4} = x^4$	<ol style="list-style-type: none"> <li>1 Use the rule <math>a^m \times a^n = a^{m+n}</math></li> <li>2 Use the rule <math>\frac{a^m}{a^n} = a^{m-n}</math></li> </ol>
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**Example 7** Write  $\frac{1}{3x}$  as a single power of  $x$

$\frac{1}{3x} = \frac{1}{3}x^{-1}$	Use the rule $\frac{1}{a^m} = a^{-m}$ , note that the fraction $\frac{1}{3}$ remains unchanged
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**Example 8** Write  $\frac{4}{\sqrt{x}}$  as a single power of  $x$

$\frac{4}{\sqrt{x}} = \frac{4}{x^{\frac{1}{2}}}$ $= 4x^{-\frac{1}{2}}$	<ol style="list-style-type: none"> <li>1 Use the rule <math>a^{\frac{1}{n}} = \sqrt[n]{a}</math></li> <li>2 Use the rule <math>\frac{1}{a^m} = a^{-m}</math></li> </ol>
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## Practice

1 Evaluate.

**a**  $14^0$

**b**  $3^0$

**c**  $5^0$

**d**  $x^0$

2 Evaluate.

**a**  $49^{\frac{1}{2}}$

**b**  $64^{\frac{1}{3}}$

**c**  $125^{\frac{1}{3}}$

**d**  $16^{\frac{1}{4}}$

3 Evaluate.

**a**  $25^{\frac{3}{2}}$

**b**  $8^{\frac{5}{3}}$

**c**  $49^{\frac{3}{2}}$

**d**  $16^{\frac{3}{4}}$

4 Evaluate.

**a**  $5^{-2}$

**b**  $4^{-3}$

**c**  $2^{-5}$

**d**  $6^{-2}$

5 Simplify.

**a**  $\frac{3x^2 \times x^3}{2x^2}$

**b**  $\frac{10x^5}{2x^2 \times x}$

**c**  $\frac{3x \times 2x^3}{2x^3}$

**d**  $\frac{7x^3y^2}{14x^5y}$

**e**  $\frac{y^2}{y^{\frac{1}{2}} \times y}$

**f**  $\frac{c^{\frac{1}{2}}}{c^2 \times c^{\frac{3}{2}}}$

**g**  $\frac{(2x^2)^3}{4x^0}$

**h**  $\frac{x^{\frac{1}{2}} \times x^{\frac{3}{2}}}{x^{-2} \times x^3}$

**Watch out!**

Remember that any value raised to the power of zero is 1. This is the rule  $a^0 = 1$ .

6 Evaluate.

**a**  $4^{-\frac{1}{2}}$

**b**  $27^{-\frac{2}{3}}$

**c**  $9^{-\frac{1}{2}} \times 2^3$

**d**  $16^{\frac{1}{4}} \times 2^{-3}$

**e**  $\left(\frac{9}{16}\right)^{-\frac{1}{2}}$

**f**  $\left(\frac{27}{64}\right)^{-\frac{2}{3}}$

7 Write the following as a single power of  $x$ .

**a**  $\frac{1}{x}$

**b**  $\frac{1}{x^7}$

**c**  $\sqrt[4]{x}$

**d**  $\sqrt[5]{x^2}$

**e**  $\frac{1}{\sqrt[3]{x}}$

**f**  $\frac{1}{\sqrt[3]{x^2}}$

8 Write the following without negative or fractional powers.

**a**  $x^{-3}$

**b**  $x^0$

**c**  $x^{\frac{1}{5}}$

**d**  $x^{\frac{2}{5}}$

**e**  $x^{-\frac{1}{2}}$

**f**  $x^{\frac{3}{4}}$

9 Write the following in the form  $ax^n$ .

**a**  $5\sqrt{x}$

**b**  $\frac{2}{x^3}$

**c**  $\frac{1}{3x^4}$

**d**  $\frac{2}{\sqrt{x}}$

**e**  $\frac{4}{\sqrt[3]{x}}$

**f** 3

## Extend

10 Write as sums of powers of  $x$ .

**a**  $\frac{x^5 + 1}{x^2}$

**b**  $x^2\left(x + \frac{1}{x}\right)$

**c**  $x^{-4}\left(x^2 + \frac{1}{x^3}\right)$

## Answers

**1 a** 1

**b** 1

**c** 1

**d** 1

**2 a** 7

**b** 4

**c** 5

**d** 2

**3 a** 125

**b** 32

**c** 343

**d** 8

**4 a**  $\frac{1}{25}$

**b**  $\frac{1}{64}$

**c**  $\frac{1}{32}$

**d**  $\frac{1}{36}$

**5 a**  $\frac{3x^3}{2}$

**b**  $5x^2$

**c**  $3x$

**d**  $\frac{y}{2x^2}$

**e**  $y^{\frac{1}{2}}$

**f**  $c^{-3}$

**g**  $2x^6$

**h**  $x$

**6 a**  $\frac{1}{2}$

**b**  $\frac{1}{9}$

**c**  $\frac{8}{3}$

**d**  $\frac{1}{4}$

**e**  $\frac{4}{3}$

**f**  $\frac{16}{9}$

**7 a**  $x^{-1}$

**b**  $x^{-7}$

**c**  $x^{\frac{1}{4}}$

**d**  $x^{\frac{2}{5}}$

**e**  $x^{-\frac{1}{3}}$

**f**  $x^{\frac{2}{3}}$

**8 a**  $\frac{1}{x^3}$

**b** 1

**c**  $\sqrt[5]{x}$

**d**  $\sqrt[5]{x^2}$

**e**  $\frac{1}{\sqrt{x}}$

**f**  $\frac{1}{\sqrt[4]{x^3}}$

**9 a**  $5x^{\frac{1}{2}}$

**b**  $2x^{-3}$

**c**  $\frac{1}{3}x^{-4}$

**d**  $2x^{-\frac{1}{2}}$

**e**  $4x^{-\frac{1}{3}}$

**f**  $3x^0$

**10 a**  $x^3 + x^{-2}$

**b**  $x^3 + x$

**c**  $x^{-2} + x^{-7}$